Dynamics of Jacobi’s elliptic spatial waves in a nonlinear optical grating

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This paper presents the results of our study on the dynamics of Jacobi’s elliptic spatial waves in a nonlinear optical grating based on a generalized coupled-mode model. We discuss the characteristics of their amplitudes, widths, and spatial periods as well as their bifurcation in the associated phase plane. Our study on the dynamical propagation of perturbed profiles reveal that these waves can suffer breathing and broadening due to the diffraction effect. A remarkable split-off phenomenon of a spatial wave with wide stripes into several narrow and shallow oblique stripes is observed, as well as their passing and bouncing collisions.

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I. INTRODUCTION

The dynamics of stationary periodic nonlinear optical waves has been studied intensively in recent years [1,2] due to its potential applications such as in optically induced photonic lattices [3] and optical interconnects [4]. One of the nonlinear effects that could generate these waves is the intensity dependent refractive index (IDRI) effect of third-order Kerr nonlinearity that changes the local refractive index of the medium [5]. The most well-studied spatial waves that exist due to this nonlinear effect are the spatial solitons in the form of localized light beam (bright soliton) and localized dark on bright light background (dark soliton) [5]. Their existence is due to the delicate balance between the self-focusing and self-defocusing effects with the diffraction effect, respectively. Meanwhile, spatial solitons that exist due to other nonlinear effects, such as the second-order and saturable nonlinearities, have also been studied previously [6,7].

For a homogeneous planar waveguide with Kerr medium, in which the associated governing equation is the nonlinear Schrödinger (NLS) equation, the simplest spatial soliton profiles are represented by the sech hyperbolic (sech) and hyperbolic (tanh) functions, respectively [5]. Meanwhile, for a nonhomogeneous planar waveguide with embedded nonlinear optical grating generally consisting of unit cells with two or more alternating layers of different Kerr materials in each unit, the existence of these bright and dark solitons has been reported in Refs. [8–11]. In those studies the corresponding NLS-type couple-mode equations considered takes into account the effect of nonlinear modulation. It was revealed that these solutions can be in the forms of symmetric or asymmetric forward and backward wave envelopes, while for certain coefficient relations one free parameter solution is admitted [8].

In principle, a periodic spatial wave can be generated by arranging isolated bright or dark spatial solitons in the form of an array. Unfortunately for the case with relatively small spatial periodicity, it is not easy to achieve a stable formation since the unwanted interactions among the individual solitons during the propagation cannot be avoided [12,13]. On the other hand, the periodic Jacobi’s elliptic functions: cn, dn, and sn, where the secant and tangent hyperbolic functions are actually the special cases of these functions [14], can be considered as the most possible stable periodic formations [1,2]. As the solutions of NLS-type equations, these periodic spatial waves have been studied extensively in optics (e.g., Refs. [1,2,15–19]).

Recently the existence of cn, dn, and sn waves in the generalized couple-mode model considered in Refs. [8–11] was briefly reported in Ref. [20]. However, to our knowledge, a detailed study regarding the dynamics of these nonlinear spatial waves in the corresponding planar waveguide gratings has never been reported. Based on this fact, our present report is focused on its dynamical characteristics and bifurcation. We investigated the characteristics of their amplitudes, widths, and spatial periods as well as considered their dynamical propagation due to small perturbation and found that they can suffer breathing and broadening due to the diffraction effect. Split-off phenomenon of wide stripe sn wave due to this linear effect is also observed along with the collisions among the new stripes.

This paper is organized as follows. In Sec. II the corresponding generalized coupling mode equations will be introduced. In Sec. III the detailed derivation of Jacobi’s elliptic solutions will be discussed. The bifurcation and characteristics of the solutions are presented in Sec. IV. Furthermore, the dynamics of perturbed spatial waves will be discussed in Sec. V. The paper ends with a summary given in Sec. VI. It should be noticed that in the ensuing discussion we specifically use the abbreviations of cn, dn, and sn waves for the commonly used nomenclatures cnoidal, dnoial, and snoidal waves, respectively.

II. GENERALIZED COUPLED-MODE MODEL

Based on the slowly varying envelope approximation (SVEA) [5], we consider the propagating transverse electric (TE) mode field in the following dimensional and complex form:

\[ E(x,y,z,t) = \Psi(y)[E_f(x,z)e^{ikz} + E_b(x,z)e^{-ikz}]e^{-iN\omega t} \delta, \]

(1)

Here \( x \) and \( z \) are the transversal and longitudinal coordinates, respectively, and the carrier wave number \( k \) is...